ECL 4340

POWER SYSTEMS

LECTURE 3
THREE-PHASE POWER

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ANNOUNCEMENT

- Please read Chapters 1 and 2
- HW 2 is due on 9/9, Friday, in Canvas.
- Office Hour for TA:

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PER PHASE ANALYSIS

- Per phase analysis allows analysis of balanced 3ϕ systems with the same effort as for a single-phase system
- Balanced 3ϕ Theorem: For a balanced 3ϕ system with
  - All loads and sources Y connected
  - No mutual Inductance between phases

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PER PHASE ANALYSIS, CONT'D

- Then
  - All neutrals are at the same potential
  - All phases are COMPLETELY decoupled
  - All system values are the same sequence as sources. The sequence order we've been using (phase b lags phase a and phase c lags phase b) is known as "positive" sequence (negative and zero sequence systems are mostly covered in Chapter 8)

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PER PHASE ANALYSIS PROCEDURE

To do per phase analysis

1. Convert all Δ load/sources to equivalent Y's
2. Solve phase "a" independent of the other phases
3. Total system power $S = 3\ V_a I_a^*$
4. If desired, phase "b" and "c" values can be determined by inspection (i.e., ±120° degree phase shifts)
5. If necessary, go back to original circuit to determine line-line values or internal Δ values.

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PER PHASE EXAMPLE

Assume a 3ϕ, Y-connected generator with $V_{an} = 1\angle 0°$ volts supplies a Δ-connected load with $Z_\Delta = -j\Omega$ through a transmission line with impedance of $j0.1\Omega$ per phase.

The load is also connected to a Δ-connected generator with $V_{a''b''} = 1\angle 0°$ through a second transmission line which also has an impedance of $j0.1\Omega$ per phase.

Find

1. The load voltage $V_{a'b'}$
2. The total power supplied by each generator, S_Y and S_Δ

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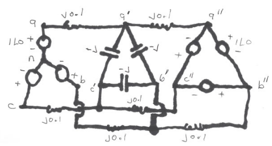
Per Phase Example, cont'd



First convert the delta load and source to equivalent Y values and draw just the "a" phase circuit

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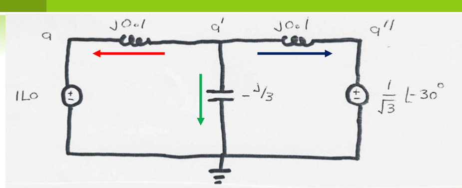
Per Phase Example, cont'd



To solve the circuit, write the KCL equation at a'

$$(V_a' - 1\angle 0)(-10j) + V_a'(3j) + (V_a' - \frac{1}{\sqrt{3}}\angle -30°)(-10j) = 0$$

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Per Phase Example, cont'd

To solve the circuit, write the KCL equation at a'

$$(V_a' - 1\angle 0)(-10j) + V_a'(3j) + (V_a' - \frac{1}{\sqrt{3}}\angle -30°)(-10j) = 0$$

$$(10j + \frac{10}{\sqrt{3}}\angle 60°) = V_a'(10j - 3j + 10j)$$

$V_a' = 0.9\angle -10.9°$ volts $\quad V_b' = 0.9\angle -130.9°$ volts

$V_c' = 0.9\angle 109.1°$ volts $\quad V_{ab}' = 1.56\angle 19.1°$ volts

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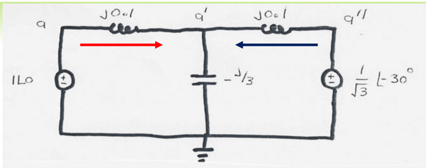
PER PHASE EXAMPLE, CONT'D



$$S_{\text{Ygen}} = 3V_a I_a^* = 3V_a\left(\frac{V_a - V_a'}{j0.1}\right)^* = 5.1 + j3.5 \text{ VA}$$

$$S_{\Delta gen} = 3V_a''\left(\frac{V_a'' - V_a'}{j0.1}\right)^* = -5.1 - j4.7 \text{ VA}$$

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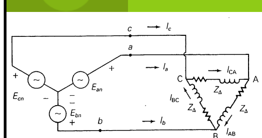
THREE-PHASE POWER



Three Phase Power:

3ϕ power is the sum of three single-phase power. Assume Y. Consider phase a voltage and current:

$$v_{an}(t) = \sqrt{2}\, V_{LN} \cos(\omega t + \delta)$$
$$i_a(t) = \sqrt{2}\, I_L \cos(\omega t + \beta)$$

Then the instantaneous power is, by definition,

$$p_a(t) = v_{an}(t)\, i_a(t) = 2V_{LN} I_L \cos(\omega t + \delta)\cos(\omega t + \beta)$$
$$= V_{LN} I_L \cos(\delta - \beta) + \underbrace{V_{LN} I_L \cos(2\omega t + \delta + \beta)}$$

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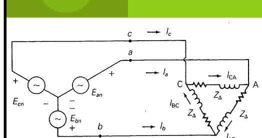
THREE-PHASE POWER



Then the instantaneous power is, by definition,

$$p_a(t) = v_{an}(t)\, i_a(t) = 2V_{LN} I_L \cos(\omega t + \delta)\cos(\omega t + \beta)$$
$$= V_{LN} I_L \cos(\delta - \beta) + \underbrace{V_{LN} I_L \cos(2\omega t + \delta + \beta)}$$

Similary for phase b, with $v_{bn}(t)$ and $i_b(t)$ shifted by -120°,

$$p_b(t) = v_{bn}(t)\, i_b(t) = 2V_{LN} I_L \cos(\omega t + \delta - 120^\circ)\cos(\omega t + \beta - 120^\circ)$$
$$= V_{LN} I_L \cos(\delta - \beta) + \underbrace{V_{LN} I_L \cos(2\omega t + \delta + \beta - 240^\circ)}$$
$$p_c(t) = v_{cn}(t)\, i_c(t) = 2V_{LN} I_L \cos(\omega t + \delta + 120^\circ)\cos(\omega t + \beta + 120^\circ)$$
$$= V_{LN} I_L \cos(\delta - \beta) + \underbrace{V_{LN} I_L \cos(2\omega t + \delta + \beta + 240^\circ)}$$

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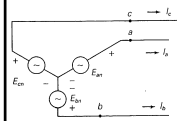
THREE-PHASE POWER



$$p_a(t) = v_{an}(t)\, i_a(t) = 2 V_{LN} I_L \cos(\omega t + \delta) \cos(\omega t + \beta)$$
$$= V_{LN} I_L \cos(\delta - \beta) + V_{LN} I_L \cos(2\omega t + \delta + \beta)$$

Similarly for phase b, with $v_{bn}(t)$ and $i_b(t)$ shifted by $-120°$,

$$p_b(t) = v_{bn}(t)\, i_b(t) = 2 V_{LN} I_L \cos(\omega t + \delta - 120°) \cos(\omega t + \beta - 120°)$$
$$= V_{LN} I_L \cos(\delta - \beta) + V_{LN} I_L \cos(2\omega t + \delta + \beta - 240°)$$

$$p_c(t) = v_{cn}(t)\, i_c(t) = 2 V_{LN} I_L \cos(\omega t + \delta + 120°) \cos(\omega t + \beta + 120°)$$
$$= V_{LN} I_L \cos(\delta - \beta) + V_{LN} I_L \cos(2\omega t + \delta + \beta + 240°)$$

Note that the second terms will add up to zero. Thus, the total instantaneous power is

$$p_{3\phi}(t) = p_a(t) + p_b(t) + p_c(t)$$
$$= \underbrace{3 V_{LN} I_L \cos(\delta - \beta)}_{\text{constant!}} + 0$$

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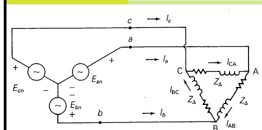
THREE-PHASE POWER



$$p_{3\phi}(t) = p_a(t) + p_b(t) + p_c(t)$$
$$= \underbrace{3 V_{LN} I_L \cos(\delta - \beta)}_{\text{constant!}} + 0$$

This implies that the 3ϕ instantaneous power is constant, which provides constant torque when applied to a motor. Thus, by definition, real power is the average power:

$$P_{3\phi} = p_{3\phi}(t) = 3 V_{LN} I_L \cos(\delta - \beta)$$

Using the line voltage $V_{LL} = \sqrt{3}\, V_{LN}$

$$P_{3\phi} = \sqrt{3}\, V_{LL} I_L \cos\theta \text{ ,} \quad \text{where } \theta = \delta - \beta$$
: power factor angle.

Note: power factor is preserved from the single-phase power factor.

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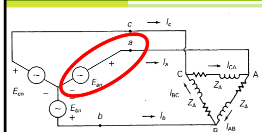
THREE-PHASE POWER



To find the reactive power we compute the complex power.

Complex Power: Consider 3ϕ voltage & current phasors:

$V_{an} = V_{LN} \angle \delta$ $\quad$ $V_{bn} = V_{LN} \angle \delta - 120°$ $\quad$ $V_{cn} = V_{LN} \angle \delta + 120°$

$I_a = I_L \angle \beta$ $\quad$ $I_b = I_L \angle \beta - 120°$ $\quad$ $I_c = I_L \angle \delta + 120°$

Then the complex power for phase a is

$$S_a = V_{an} I_a^* = V_{LN} I_L \angle \delta - \beta$$

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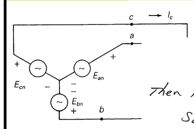
THREE-PHASE POWER



$V_{an} = V_{LN}\angle\delta$ $\quad V_{bn} = V_{LN}\angle\delta-120^\circ$ $\quad V_{cn} = V_{LN}\angle\delta+120^\circ$

$I_a = I_L\angle\beta$ $\quad I_b = I_L\angle\beta-120^\circ$ $\quad I_c = I_L\angle\delta+120^\circ$

Then the complex power for phase a is

$$S_a = V_{an} I_a^* = V_{LN} I_L \angle\delta-\beta$$

But, we also see that

$$S_b = V_{bn} I_b^* = V_{LN} I_L \angle\delta-\beta = S_c = S_a$$

Thus, the total complex power is

$$S_{3\phi} = S_a + S_b + S_c = 3 S_a$$
$$= 3 V_{LN} I_L \angle\delta-\beta$$
$$= \underbrace{3 V_{LN} I_L \cos(\delta-\beta)}_{P_{3\phi}} + j\,\underbrace{3 V_{LN} I_L \sin(\delta-\beta)}_{Q_{3\phi}}$$

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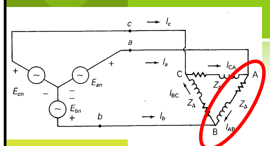
THREE-PHASE POWER



These expressions are also the same in <u>Δ load</u>:

Consider Δ circuit with the line voltage and Δ current:

$$E_{ab} = V_{LL}\angle\delta \;,\quad I_{ab} = I_\Delta \angle\beta$$

Then, the complex power for a load between line a&b is

$$S_{ab} = E_{ab} I_{ab}^* = V_{LL} I_\Delta \angle\delta-\beta$$

But, we also see that

$$S_{bc} = E_{bc} I_{bc}^* = V_{LL} I_\Delta \angle\delta-\beta = S_{ca} = S_{ab}$$

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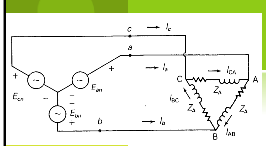
THREE-PHASE POWER



Thus, the total complex power for Δ load is:

$$S_{3\phi} = S_{ab} + S_{bc} + S_{ca} = 3 S_{ab}$$
$$= 3 V_{LL} I_\Delta \angle\delta-\beta$$
$$= \sqrt{3} V_{LL} I_L \angle\delta-\beta, \quad \text{where } I_L = \sqrt{3} I_\Delta$$

Therefore,

$$P_{3\phi} = \sqrt{3} V_{LL} I_L \cos(\delta-\beta)$$
$$Q_{3\phi} = \sqrt{3} V_{LL} I_L \sin(\delta-\beta)$$
$$|S_{3\phi}| = \sqrt{3} V_{LL} I_L$$

<u>Note</u>: For both Y and Δ loads, the power expressions are identical, <u>provided that</u> we use <u>line voltage</u> V_{LL} and <u>line current</u> I_L.

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Three Phase Transmission Line

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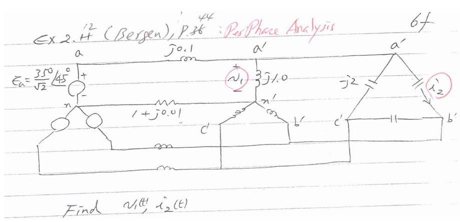
Three Phase Examples



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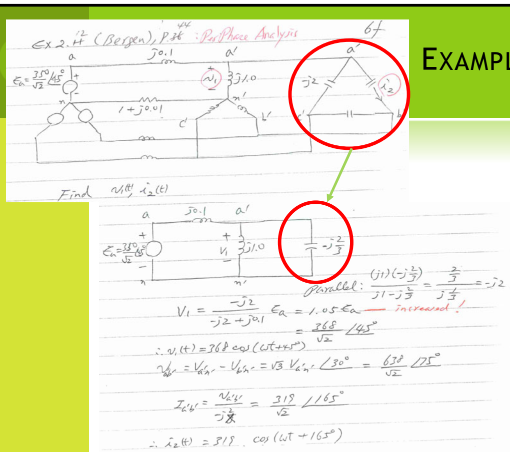
Examples



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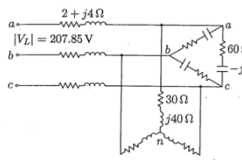
Three Phase Examples

Example 2.7

A three-phase line has an impedance of $2 + j4\ \Omega$ as shown



Taking the phase voltage V_a as reference, determine:

(a) The current, real power, and reactive power drawn from the supply.

(b) The line voltage at the combined loads.

(c) The current per phase in each load.

(d) The total real and reactive powers in each load and the line.

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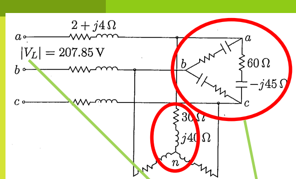
Three Phase Examples



(a) The Δ-connected load is transformed into an equivalent Y. The impedance per phase of the equivalent Y is

$$Z_2 = \frac{60 - j45}{3} = 20 - j15\ \ \Omega$$

The phase voltage is

$$V_1 = \frac{207.85}{\sqrt{3}} = 120\ \text{V}$$

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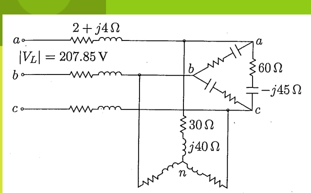
Three Phase Examples



The single-phase equivalent circuit is shown in Figure 2.20.

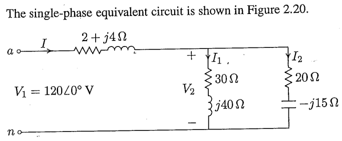


FIGURE 2.20

Single-phase equivalent circuit for Example 2.7.

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Three Phase Examples

The single-phase equivalent circuit is shown in Figure 2.20.

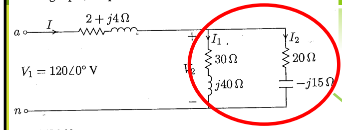

FIGURE 2.20
Single-phase equivalent circuit for Example 2.7.

The total impedance is

$$Z = 2 + j4 + \frac{(30 + j40)(20 - j15)}{(30 + j40) + (20 - j15)}$$
$$= 2 + j4 + 22 - j4 = 24\ \Omega$$

With the phase voltage V_{an} as reference, the current in phase a is

$$I = \frac{V_1}{Z} = \frac{120\angle 0°}{24} = 5\ \text{A}$$

The three-phase power supplied is

$$S = 3V_1I^* = 3(120\angle 0°)(5\angle 0°) = 1800\ \text{W}$$

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Three Phase Examples

The single-phase equivalent circuit is shown in Figure 2.20.

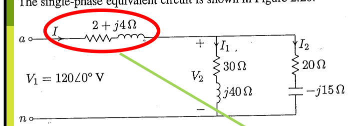

FIGURE 2.20
Single-phase equivalent circuit for Example 2.7.

(b) The phase voltage at the load terminal is

$$V_2 = 120\angle 0° - (2 + j4)(5\angle 0°) = 110 - j20$$
$$= 111.8\angle{-10.3°}\ \text{V}$$

The line voltage at the load terminal is

$$V_{2ab} = \sqrt{3}\angle 30°\ V_2 = \sqrt{3}\ (111.8)\angle 19.7° = 193.64\angle 19.7°\ \text{V}$$

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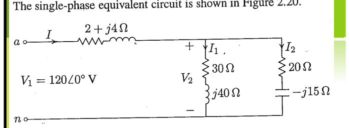
Three Phase Examples

The single-phase equivalent circuit is shown in Figure 2.20.



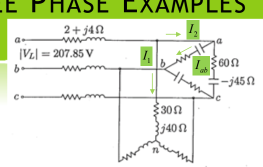


FIGURE 2.20
Single-phase equivalent circuit for Example 2.7.

(c) The current per phase in the Y-connected load and in the equivalent Y of the Δ load is

$$I_1 = \frac{V_2}{Z_1} = \frac{110 - j20}{30 + j40} = 1 - j2 = 2.236\angle - 63.4°\ \text{A}$$
$$I_2 = \frac{V_2}{Z_2} = \frac{110 - j20}{20 - j15} = 4 + j2 = 4.472\angle 26.56°\ \text{A}$$

The phase current in the original Δ-connected load, i.e., I_{ab} is given by

$$I_{ab} = \frac{I_2}{\sqrt{3}\angle{-30°}} = \frac{4.472\angle 26.56°}{\sqrt{3}\angle{-30°}} = 2.582\angle 56.56°\ \text{A}$$

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Three Phase Examples

The single-phase equivalent circuit is shown in Figure 2.20.

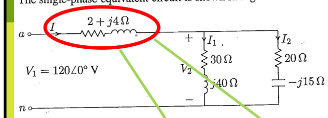

FIGURE 2.20
Single-phase equivalent circ

(d) The three-phase power absorbed by each load is

$$S_1 = 3V_2 I_1^* = 3(111.8\angle -10.3°)(2.236\angle 63.4°) = 450 \text{ W} + j600 \text{ var}$$
$$S_2 = 3V_2 I_2^* = 3(111.8\angle -10.3°)(4.472\angle -26.56°) = 1200 \text{ W} - j900 \text{ var}$$

The three-phase power absorbed by the line is

$$S = VI^* = (ZI)I^* = Z|I|^2, \quad V = V_1 - V_2$$
$$= R|I|^2 + jX|I|^2$$

$$S_L = 3(R_L + jX_L)|I|^2 = 3(2 + j4)(5)^2 = 150 \text{ W} + j300 \text{ var}$$

$$S = VI^* = \frac{VV^*}{Z^*} = \frac{|V|^2}{Z^*}$$

It is clear that the sum of load powers and line losses is equal to the power delivered from the supply, i.e.,

$$S_1 + S_2 + S_L = (450 + j600) + (1200 - j900) + (150 + j300)$$
$$= 1800 \text{ W} + j0 \text{ var}$$

$$\left(\text{or } Z = \frac{|V|^2}{S^*}\right)$$

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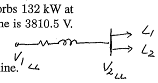
Three Phase Examples

Example 2.8

A three-phase line has an impedance of $0.4 + j2.7\ \Omega$ per phase. The line feeds two balanced three-phase loads that are connected in parallel. The first load is absorbing 560.1 kVA at 0.707 power factor lagging. The second load absorbs 132 kW at unity power factor. The line-to-line voltage at the load end of the line is 3810.5 V. Determine:



(a) The magnitude of the line voltage at the source end of the line.
(b) Total real and reactive power loss in the line.
(c) Real power and reactive power supplied at the sending end of the line.

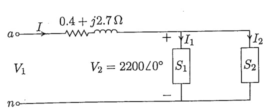


FIGURE 2.21
Single-phase equivalent diagram for Example 2.8.

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Three Phase Examples

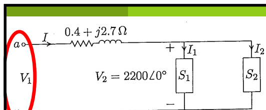

FIGURE 2.21
Single-phase equivalent diagram

(a) The phase voltage at the load terminals is

$$V_2 = \frac{3810.5}{\sqrt{3}} = 2200 \text{ V}$$

The single-phase equivalent circuit is shown in Figure 2.21.
The total complex power is

$$S_{R(3\phi)} = \underbrace{560.1(0.707 + j0.707)}_{S_1} + \underbrace{132}_{S_2} = 528 + j396$$
$$= 660\angle 36.87° \text{ kVA}$$

With the phase voltage V_2 as reference, the current in the line is $S = 3VI^*$

$$I = \frac{S^*_{R(3\phi)}}{3V_2^*} = \frac{660{,}000\angle -36.87°}{3(2200\angle 0°)} = 100\angle -36.87° \text{ A}$$

The phase voltage at the sending end is

$$V_1 = 2200\angle 0° + (0.4 + j2.7)100\angle -36.87° = 2401.7\angle 4.58° \text{ V}$$

The magnitude of the line voltage at the sending end of the line is

$$|V_{1L}| = \sqrt{3}|V_1| = \sqrt{3}(2401.7) = 4160 \text{ V}$$

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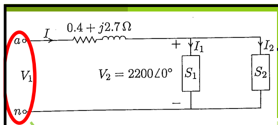

FIGURE 2.21
Single-phase equivalent diagr

Three Phase Examples

The phase voltage at the sending end is

$$V_1 = 2200\angle 0^\circ + (0.4 + j2.7)100\angle -36.87^\circ = 2401.7\angle 4.58^\circ \text{ V}$$

The magnitude of the line voltage at the sending end of the line is

$$|V_{1L}| = \sqrt{3}|V_1| = \sqrt{3}(2401.7) = 4160 \text{ V}$$

(b) The three-phase power loss in the line is $S = VI^* = (ZI)I^* = Z|I|^2$

$$S_{L(3\phi)} = 3R|I|^2 + j3X|I|^2 = 3(0.4)(100)^2 + j3(2.7)(100)^2$$
$$= 12 \text{ kW} + j81 \text{ kvar}$$

(c) The three-phase sending power is

$$S_{S(3\phi)} = 3V_1 I^* = 3(2401.7\angle 4.58^\circ)(100\angle 36.87^\circ) = 540 \text{ kW} + j477 \text{ kvar}$$

It is clear that the sum of load powers and the line losses is equal to the power delivered from the supply, i.e.,

$$S_{S(3\phi)} = S_{R(3\phi)} + S_{L(3\phi)} = (528 + j396) + (12 + j81) = 540 \text{ kW} + j477 \text{ kvar}$$

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